1. [Maximum mark: 5] SPM.1.SL.TZ0.2

Let A and B be events such that $\mathrm{P}(A) = 0.5, \mathrm{P}(B) = 0.4$ and $\mathrm{P}(A \cup B) = 0.6$.

Find $\mathrm{P}(A|B)$. [5]

Markscheme

attempt to substitute into $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ ***(M1)***

Note: Accept use of Venn diagram or other valid method.

$0.6 = 0.5 + 0.4 - \mathrm{P}(A \cap B)$ ***(A1)***

$\mathrm{P}(A \cap B) = 0.3$ (seen anywhere) ***A1***

attempt to substitute into $\mathrm{P}(A|B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$ ***(M1)***

$= \frac{0.3}{0.4}$

$\mathrm{P}(A|B) = 0.75 \left(= \frac{3}{4}\right)$ ***A1***

[5 marks]

[Maximum mark: 5] 23M.1.SL.TZ2.4

Events A and B are such that $\mathrm{P}(A) = 0.4, \mathrm{P}(A|B) = 0.25$ and $\mathrm{P}(A \cup B) = 0.55$.

Find $\mathrm{P}(B)$. [5]

Markscheme

substitutes into $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ to form

$0.55 = 0.4 + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ (or equivalent) ***(A1)***

substitutes into $\mathrm{P}\left(A\middle|B\right) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$ to form $0.25 = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$ (or equivalent) ***(A1)***

attempts to combine their two probability equations to form an equation in $\mathrm{P}(B)$ ***(M1)***

Note: The above two ***A*** marks are awarded independently.

correct equation in $\mathrm{P}(B)$ ***A1***

$0.55 = 0.4 + \mathrm{P}(B) - 0.25\mathrm{P}(B)$ OR $\frac{\mathrm{P}(B) - 0.15}{\mathrm{P}(B)} = 0.25$ OR $\mathrm{P}(B) - 0.15 = 0.25\mathrm{P}(B)$

(or equivalent)

$\mathrm{P}(B) = \frac{15}{75} \left(= \frac{1}{5} = 0.2\right)$ ***A1***

[5 marks]

12. [Maximum mark: 6] 22M.2.SL.TZ1.6

Let A and B be two independent events such that $\mathrm{P}(A \cap B') = 0.16$ and $\mathrm{P}(A' \cap B) = 0.36$.

(a) Given that $\mathrm{P}(A \cap B) = x$, find the value of x. [4]

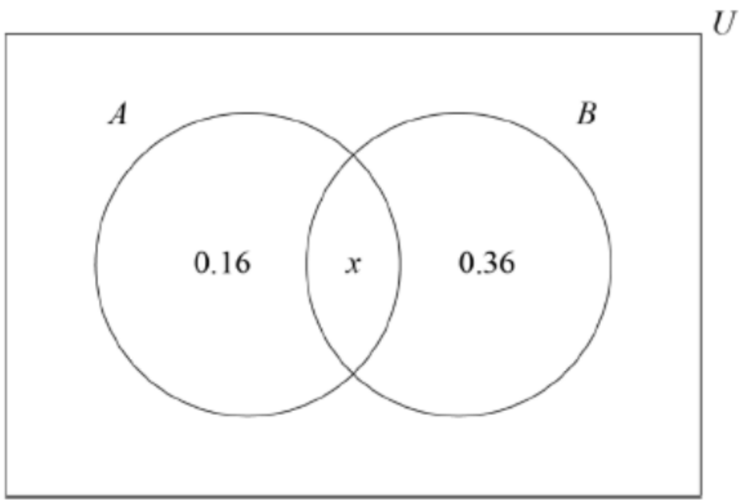
Markscheme

METHOD 1

EITHER

one of $\mathrm{P}(A) = x + 0.16$ OR $\mathrm{P}(B) = x + 0.36$ ***A1***

OR



A1

THEN

attempt to equate their $\mathrm{P}(A \cap B)$ with their expression for $\mathrm{P}(A) \times \mathrm{P}(B)$ ***M1***

$\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B) \Rightarrow x = (x + 0.16) \times (x + 0.36)$ ***A1***

$x = 0.24$ ***A1***

METHOD 2

attempt to form at least one equation in $\mathrm{P}(A)$ and $\mathrm{P}(B)$ using independence ***M1***

$(\mathrm{P}(A \cap B\prime) = \mathrm{P}(A) \times P(B\prime) \Rightarrow)\ \mathrm{P}(A) \times (1 - \mathrm{P}(B)) = 0.16$

OR

$(\mathrm{P}(A\prime \cap B) = \mathrm{P}(A\prime) \times P(B) \Rightarrow)\ (1 - \mathrm{P}(A)) \times \mathrm{P}(B) = 0.36$

$\mathrm{P}(A) = 0.4$ and $\mathrm{P}(B) = 0.6$ ***A1***

$\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B) = 0.4 \times 0.6$ ***(A1)***

$x = 0.24$ ***A1***

[4 marks]

(b) Find $\mathrm{P}(A\prime \mid B\prime)$. [2]

Markscheme

METHOD 1

recognising $\mathrm{P}(A\prime \mid B\prime) = \mathrm{P}(A\prime)$ ***(M1)***

$= 1 - 0.16 - 0.24$

$= 0.6$ ***A1***

METHOD 2

$\mathrm{P}(B) = 0.36 + 0.24 (= 0.6)$

$\mathrm{P}(A\prime \mid B\prime) = \frac{\mathrm{P}(A\prime \cap B\prime)}{\mathrm{P}(B\prime)}\ \left(= \frac{0.24}{0.4}\right)$ ***(A1)***

$= 0.6$ ***A1***

[2 marks]

[Maximum mark: 6] 22M.2.SL.TZ2.4

Events A and B are independent and $\mathrm{P}(A) = 3\mathrm{P}(B)$.

Given that $\mathrm{P}(A \cup B) = 0.68$, find $\mathrm{P}(B)$. [6]

Markscheme

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B) = 0.68$

substitution of $\mathrm{P}(A) \cdot \mathrm{P}(B)$ for $\mathrm{P}(A \cap B)$ in $\mathrm{P}(A \cup B)$ ***(M1)***

$\mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A)\mathrm{P}(B) \ (= 0.68)$

substitution of $3\mathrm{P}(B)$ for $\mathrm{P}(A)$ ***(M1)***

$3\mathrm{P}(B) + \mathrm{P}(B) - 3\mathrm{P}(B)\mathrm{P}(B) = 0.68$ (or equivalent) ***(A1)***

Note: The first two ***M*** marks are independent of each other.

attempts to solve their quadratic equation ***(M1)***

$\mathrm{P}(B) = 0.2,\ 1.133\ldots\ \left(\frac{1}{5},\ \frac{17}{15}\right)$

$\mathrm{P}(B) = 0.2\ \left(= \frac{1}{5}\right)$ ***A2***

Note: Award ***A1*** if both answers are given as final answers for $\mathrm{P}(B)$.

[6 marks]

[Maximum mark: 5] 21N.1.SL.TZ0.4

Box 1 contains 5 red balls and 2 white balls.

Box 2 contains 4 red balls and 3 white balls.

(a) A box is chosen at random and a ball is drawn. Find the probability that the ball is red. [3]

Markscheme

valid approach to find $\mathrm{P}(R)$ ***(M1)***

tree diagram (must include probabilty of picking box) with correct required probabilities

OR $\mathrm{P}(R \cap B_1) + \mathrm{P}(R \cap B_2)$ OR $\mathrm{P}(R|\, B_1)\mathrm{P}(B_1) + \mathrm{P}(R|\, B_2)\mathrm{P}(B_2)$

$\frac{5}{7} \cdot \frac{1}{2} + \frac{4}{7} \cdot \frac{1}{2}$ ***(A1)***

$\mathrm{P}(R) = \frac{9}{14}$ ***A1***

[3 marks]

(b) Let A be the event that "box 1 is chosen" and let R be the event that "a red ball is drawn".

Determine whether events A and R are independent. [2]

Markscheme

events A and R are not independent, since $\frac{9}{14} \cdot \frac{1}{2} \neq \frac{5}{14}$ OR $\frac{5}{7} \neq \frac{9}{14}$ OR $\frac{5}{9} \neq \frac{1}{2}$

OR an explanation e.g. different number of red balls in each box ***A2***

Note: Both conclusion and reasoning are required. Do not split the ***A2***.

[Maximum mark: 8] 21M.2.SL.TZ2.4

At a school, 70% of the students play a sport and 20% of the students are involved in theatre. 18% of the students do neither activity.

A student is selected at random.

(a) Find the probability that the student plays a sport and is involved in theatre. [2]

Markscheme

EITHER

$\mathrm{P}(S) + \mathrm{P}(T) + \mathrm{P}(S' \cap T') - \mathrm{P}(S \cap T) = 1$ OR $\mathrm{P}(S \cup T) = \mathrm{P}((S' \cap T')')$ ***(M1)***

$0.7 + 0.2 + 0.18 - \mathrm{P}(S \cap T) = 1$ OR $\mathrm{P}(S \cup T) = 1 - 0.18$

OR

a clearly labelled Venn diagram ***(M1)***

THEN

$P(S \cap T) = 0.08$ (accept 8%) ***A1***

Note: To obtain the ***M1*** for the Venn diagram all labels must be correct and in the correct sections. For example, do not accept 0.7 in the area corresponding to $S \cap T'$.

[2 marks]

(b) Find the probability that the student is involved in theatre, but does not play a sport. [2]

Markscheme

EITHER

$\mathrm{P}(T \cap S\prime) = \mathrm{P}(T) - \mathrm{P}(T \cap S)(= 0.2 - 0.08)$ OR

$\mathrm{P}(T \cap S\prime) = \mathrm{P}(T \cup S) - \mathrm{P}(S)(= 0.82 - 0.7)$ ***(M1)***

OR

a clearly labelled Venn diagram including $\mathrm{P}(S)$, $\mathrm{P}(T)$ and $\mathrm{P}(S \cap T)$ ***(M1)***

THEN

$= 0.12$ (accept 12%) ***A1***

[2 marks]

At the school 48% of the students are girls, and 25% of the girls are involved in theatre.

A student is selected at random. Let G be the event "the student is a girl" and let T be the event "the student is involved in theatre".

(c) Find $\mathrm{P}(G \cap T)$. [2]

Markscheme

$\mathrm{P}(G \cap T) = \mathrm{P}(T/G)\mathrm{P}(G) \ \ (0.25 \times 0.48)$ ***(M1)***

$= 0.12$ ***A1***

[2 marks]

(d) Determine if the events G and T are independent. Justify your answer. [2]

Markscheme

METHOD 1

$\mathrm{P}(G) \times \mathrm{P}(T)(= 0.48 \times 0.2) = 0.096$ ***A1***

$\mathrm{P}(G) \times \mathrm{P}(T) \neq \mathrm{P}(G \cap T) \Rightarrow G$ and T are not independent ***R1***

METHOD 2

$\mathrm{P}(T | G) = 0.25$ ***A1***

$\mathrm{P}(T | G) \neq \mathrm{P}(T) \Rightarrow G$ and T are not independent ***R1***

Note: Do not award ***A0R1***.

[2 marks]