

Name: \_\_\_\_\_

Show work needed to justify your answer.

Date: \_\_\_\_\_

HW: # 18a: Math IBSL - Standard 18 - Arithmetic and Geometric Sequences

5 points

1 For the following sequences, determine whether they are geometric and if so, find the term indicated.

a  $9, 27, 81, \dots u_6$   $r=3$

$$u_6 = 9 \cdot 3^{6-1}$$

$$u_6 = 9 \cdot 3^5 \rightarrow \boxed{u_6 = 2187}$$

d  $-4, 6, -9, \dots u_8$

$$\frac{6}{-4} = \frac{-3}{2}, \quad \frac{-9}{6} = \frac{-3}{2}$$

$$u_8 = -4 \cdot \left(\frac{-3}{2}\right)^{8-1}$$

$$u_8 = -4 \cdot \left(\frac{-3}{2}\right)^7 \rightarrow \boxed{u_8 = \frac{2187}{32}}$$

3 Write the first five terms of a geometric sequence in which the sixth term is 64.

$$u_6 = 64$$

$$\boxed{2, 4, 8, 16, 32}$$

one possible way

$$\begin{cases} 64 = u_1 \cdot r^{6-1} \\ 64 = u_1 \cdot r^5 \\ 64 = u_1 \cdot 2^5 \\ 64 = 32u_1 \rightarrow u_1 = 2 \end{cases} \quad \text{let } r=2$$

1 If a geometric sequence has  $u_5 = 40$  and  $u_{10} = 303.75$ , find the value of  $u_{15}$ .

$$40 = u_1 \cdot r^4 \quad \text{and} \quad 303.75 = u_1 \cdot r^9$$

$$\frac{40}{r^4} = u_1 \quad \frac{303.75}{r^9} = u_1$$

$$\frac{40}{r^4} = \frac{303.75}{r^9}$$

$$40r^9 = 303.75r^4$$

$$40r^5 = 303.75$$

$$r^5 = 7.59375$$

$$r = 1.5$$

$$40 = u_1 \cdot 1.5^4$$

$$40 = 5.0625 u_1$$

$$u_1 = 7.901$$

$$u_{15} = 7.901 \cdot 1.5^{14}$$

$$\boxed{u_{15} = 2306.6}$$

2 For a geometric sequence that has  $r = -\frac{4}{5}$  and  $u_6 = -1280$ , find the 20th term.

$$u_6 = u_1 \cdot \left(-\frac{4}{5}\right)^5$$

$$-1280 = -0.32768 u_1$$

$$u_1 = 3906.25$$

$$u_{20} = 3906.25 \cdot \left(-\frac{4}{5}\right)^{19}$$

$$\boxed{u_{20} = -56.3}$$

Name: \_\_\_\_\_

Show work needed to justify your answer.

Date: \_\_\_\_\_

HW: # 18a: Math IBSL - Standard 18 - Arithmetic and Geometric Sequences

5 points

3 If 16,  $x+2$ , 1 are the first three terms of a geometric sequence, find all possible values of  $x$ .

$$\begin{array}{l}
 1 = 16 \cdot r^{3-1} \quad ; \quad x+2 = 16 \cdot \frac{1}{4}^{2-1} \\
 1 = 16 \cdot r^2 \quad \quad ; \quad x+2 = 16 \cdot \frac{1}{4} \\
 \frac{1}{16} = r^2 \quad \quad \quad ; \quad x+2 = 4 \\
 \boxed{\frac{1}{4} = r} \quad \quad \quad ; \quad \boxed{x=2} \\
 \vdots \\
 \vdots
 \end{array}
 \quad \text{or} \quad
 \begin{array}{l}
 x+2 = 16 \left(\frac{1}{4}\right)^{2-1} \\
 x+2 = 16 \left(\frac{1}{4}\right) \\
 x+2 = -4 \\
 \boxed{x=-6}
 \end{array}$$

4 Find the number of terms in the geometric sequence

$r=2$

6, 12, 24, ..., 1536

$$\begin{array}{l}
 1536 = 6 \cdot 2^{n-1} \\
 256 = 2^{n-1} \\
 \ln 256 = \ln 2^{n-1} \\
 \ln 256 = (n-1) \ln 2
 \end{array}
 \quad \rightarrow \quad
 \begin{array}{l}
 n-1 = \frac{\ln 256}{\ln 2} \\
 n-1 = 8 \\
 \boxed{n=9}
 \end{array}$$

7 An old legend states that a peasant won a reward from a king. The peasant asked to be paid in rice; one grain on the first square of a chessboard, two grains on the second, four on the third square, and so on.

$$\begin{array}{l}
 u_1 = 1 \quad u_2 = 2 \quad u_3 = 4 \\
 1, 2, 4 \quad r=2
 \end{array}$$

- a How many grains of rice would be on the 30th square?
- b Which square would contain exactly 512 grains of rice?

$$\begin{array}{l}
 u_{30} = 1 \cdot 2^{30-1} \\
 u_{30} = 2^{29} \\
 \boxed{u_{30} = 538670912}
 \end{array}$$

(b)

$$\begin{array}{l}
 512 = 1 \cdot 2^{n-1} \\
 \ln 512 = (n-1) \ln 2 \\
 n-1 = \frac{\ln 512}{\ln 2} \rightarrow n-1 = 9 \rightarrow \boxed{n=10}
 \end{array}$$