Name: ______________________ **Show work needed to justify your answer.** Date: __________

HW: # 19b: Math IBSL - Standard 19 - Arithmetic and Geometric Series 5 points

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1 In an arithmetic sequence, the first term is −8 and the sum of the first 20 terms is 790.

a Find the common difference.

b **i** Find u_{28}.

ii Hence, find S_{28}.

c Find how many terms it takes for the sum to exceed 2000.

(a)

$u_{20} = d(20-1) - 8$ AND $790 = \frac{20}{2}(-8 + u_{20})$

$790 = 10(u_{20} - 8)$

$79 = u_{20} - 8$

$u_{20} = 87$

$87 = 19d - 8$

$95 = 19d$

$\boxed{d = 5}$

b) $u_{28} = 5(27) - 8$

i $\boxed{u_{28} = 127}$

(ii) $S_{28} = 14(-8 + 127)$

$\boxed{S_{28} = 1666}$

c) $2000 < \frac{n}{2}[-16 + 5(n-1)]$

$2000 < \frac{n}{2}(5n - 21)$

$\frac{5}{2}n^2 - \frac{21}{2}n - 2000 > 0$

y_1 y_2

$n = 30.7$

$\boxed{\text{So } n = 31}$ is when sum exceeds 2000.

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2 In an arithmetic series, $S_{40} = 1900$ and $u_{40} = 106$. Find the value of the first term and the common difference.

-40

$106 = 39d + u_1$

$1900 = 20(2u_1 + 39d)$

$1900 = 40u_1 + 780d$

$780d + 40u_1 = 1900$

$+ (-1560d - 40u_1 = -4240)$

$-780d = -2340$

$d = 3$

$106 = 39(3) + u_1$

$106 = 117 + u_1$

$\boxed{u_1 = -11}$

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3 The sum of an infinite geometric series is 20, and the common ratio is 0.2. Find the first term of this series.

$S_\infty = \frac{u_1}{1 - r}$

$\frac{20}{1} = \frac{u_1}{1 - 0.2}$

$\frac{20}{1} = \frac{u_1}{0.8}$

$\boxed{u_1 = 16}$

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4 The sum of an infinite geometric series is three times the first term. Find the common ratio of this series.

$S_\infty = 3u_1$

$S_\infty = \frac{u_1}{1-r}$

$\frac{3u_1}{1} = \frac{u_1}{1-r}$

$\frac{3u_1(1-r)}{u_1} = \frac{u_1}{u_1}$

$3(1-r) = 1$

$3-3r = 1$

$-3r = -2$

$r = \frac{2}{3}$

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6 In a geometric sequence, the fourth term is 8 times the first term. The sum of the first 10 terms is 2557.5. Find the 10th term of this sequence.

$u_4 = 8u_1 \quad S_{10} = 2557.5$

① $u_n = u_1 \cdot r^{n-1}$

$\frac{8u_1}{u_1} = \frac{u_1 \cdot r^3}{u_1}$

$8 = r^3$

$r = 2$

② $2557.5 = \frac{u_1(2^{10}-1)}{2-1}$

$2557.5 = 1023u_1$

$\boxed{u_1 = 2.5}$

③ $u_{10} = (2.5)(2)^{10-1}$

$u_{10} = (2.5)(2^9)$

$u_{10} = (2.5)(512)$

$\boxed{u_{10} = 1280}$

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8 A geometric sequence has all positive terms. The sum of the first two terms is 15 and the sum to infinity is 27.

a Find the value of the common ratio.

b Hence, find the first term.

$u_1 + u_2 = 15 \quad S_\infty = 27$

$u_2 = 15 - u_1$

(a) $27 = \frac{u_1}{1-r}$ And $15 - u_1 = u_1 \cdot r^1$

$15 = u_1 + u_1 r$

$15 = u_1(1+r)$

$\boxed{\frac{15}{1+r}} = u_1$

$\frac{27}{1} = \frac{\left(\frac{15}{1+r}\right)}{1-r}$

$\frac{(27-27r)}{1} = \frac{15}{1+r}$

$(27-27r)(1+r) = 15$

$u_1 \quad u_2$

$\boxed{r = \frac{2}{3}}$

(b) $u_1 = \frac{15}{1+\frac{2}{3}}$

$u_1 = \frac{\left(\frac{15}{1}\right)}{\left(\frac{5}{3}\right)}$

$u_1 = \left(\frac{15}{1}\right)\left(\frac{3}{5}\right)$

$\boxed{u_1 = 9}$